Growth and singularity in 2D fluids

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The question of global regularity remains open for many fundamental models of fluid dynamics. In two dimensions, solutions to the incompressible Euler equations have been known to be globally regular since the 1930s, although their derivatives can grow double-exponentially with time. On the other hand, this question has not yet been resolved for the more singular surface quasi-geostrophic (SQG) equation, which is used in atmospheric models. The latter state of affairs is also true for the modified SQG equations, a family of PDE which interpolate between these two models.

I will discuss two results about the patch dynamics version of these equations on the halfplane. The first is global-in-time regularity for the Euler patch model, even if the patches initially touch the boundary of the half-plane. The second is local-in-time regularity for those modified SQG patch equations that are slightly more singular than Euler, but also existence of their solutions which blow up in finite time.